and the result is

$$u - u_1 = \int_{p_1}^{p} \mathrm{d}p/\varrho c \,. \tag{19a}$$

The equivalent relation for backward-facing rarefactions is

$$u - u_1 = -\int_{p_1}^{p} \mathrm{d}p/\varrho c \,. \tag{19b}$$

Defining the new variable

$$l = \int^{p} \mathrm{d}p/\varrho c \,, \tag{20}$$

Equations (19) can be written

$$u - l = \text{const} = u_1 - l_1 \equiv -2s_1 \tag{21}$$

for forward-facing rarefactions, and

$$u + l = \text{const} = u_1 + l_1 = 2r_1 \tag{22}$$

for backward-facing ones. The constants r_1 and s_1 are called « Riemann invariants ».

To understand the implications of eqs. (21) and (22) more fully, consider the special case of isentropic flow, for which eq. (3) reduces to $\mathrm{d}S/\mathrm{d}t = 0$. Then eqs. (1) and (2) can be combined to form an equivalent set of « characteristic equations »:

$$[\partial/\partial t + (u + c)\,\partial/\partial x](u + l) = 0 \,, \tag{23}$$

$$[\partial/\partial t + (u - c)\,\partial/\partial x](u - l) = 0 \,. \tag{24}$$

The curves on which $\mathrm{d}x/\mathrm{d}t = u \pm c$ are called « characteristic curves »; eqs. (23) and (24) are thus equivalent to the set:

$$C+: \quad u + l = \text{const} = 2r\,; \qquad \mathrm{d}x/\mathrm{d}t = u + c \,, \tag{25}$$

$$C-: \quad u - l = \text{const} = -2s\,; \qquad \mathrm{d}x/\mathrm{d}t = u - c \,. \tag{26}$$

In words, r is constant on the $C+$ characteristic curves for which $\mathrm{d}x/\mathrm{d}t = u + c$; s is constant on the $C-$ characteristics. Equations (25) and (26) are true when the flow is a mixture of forward- and backward-facing waves. For forward-facing waves, eq. (21) is also true; then

$$C+: \quad u + l = 2r\,, \tag{27}$$

$$C-: \quad u - l = 2s_1\,. \tag{28}$$

For backward-facing waves:

$$C+: \quad u + l = 2r_1\,, \tag{29}$$

$$C-: \quad u - l = -2s\,. \tag{30}$$

To summarize:

Forward-facing shock waves running into material with velocity u_0 and in the state (p_0, V_0) satisfy the relations

$$D - u_0 = V_0\sqrt{(p_1 - p_0)/(V_0 - V_1)}\,, \tag{31}$$

$$u_1 - u_0 = \sqrt{(p_1 - p_0)\,(V_0 - V_1)}\,. \tag{32}$$

Backward-facing shocks, the relations

$$D - u_0 = -V_0\sqrt{(p_1 - p_0)/(V_0 - V_1)}\,, \tag{33}$$

$$u_1 - u_0 = -\sqrt{(p_1 - p_0)\,(V_0 - V_1)}\,. \tag{34}$$

For forward-facing rarefactions running into material in the uniform state (p_0, V_0) with velocity u_0,

$$u - u_0 = l - l_0\,. \tag{35}$$

For backward-facing rarefactions moving into the same state,

$$u - u_0 = -(l - l_0)\,. \tag{36}$$

For rarefactions there are no equations analogous to (31) and (33) because there is no single propagation velocity associated with a rarefaction. Waves for which eq. (35) or eq. (36) applies are called « simple waves ». In the general case of isentropic flow in which, for example, two rarefactions are interacting, eqs. (25) and (26) describe the flow.

In condensed materials compressed by shock waves to about 15 % of their initial volume or less, the shock wave is called « weak ». Then the entropy-